Lines Summary Sheet

Mid-Point Between Two Points

Given two points the mid-point is just the average of the x's and y's. So the mid-point between (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Therefore mid-point of $(3, 5)$ and $(7, -1)$ is $(5, 2)$.

Length Between Two Points

Given two points in the xy-plane (x_1, y_1) and (x_2, y_2) then the length (distance) between them is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This is derived from Pythagoras' applied to a right angled triangle between the points.

Definition Of Gradient

The gradient between two points is defined to be the difference in y divided by difference in x. Therefore between (x_1, y_1) and (x_2, y_2) it is

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Equation Of Line From A Point And A Gradient

Given a gradient and a point you can work out the equation of the line. You substitute the point into the equation $y = mx + c$ and solve for c. For example find the equation of the line of gradient 3 through the point $(12, -5)$. Well $y = 3x + c$ so placing $(12, -5)$ in we find

$$\begin{aligned} y &= 3x + c \\ -5 &= 3 \times 12 + c \\ -41 &= c. \end{aligned}$$

So the line is $y = 3x - 41$.

Equation Of Line From Two Points

Given two points we can find the equation of the line as follows. Firstly find the gradient between the points and then follow the same procedure as above. For example find the equation of the line through $(3, -4)$ and $(6, 7)$. The gradient is $\frac{7-(-4)}{6-3} = \frac{11}{3}$. Therefore the line is $y = \frac{11}{3}x + c$ through both the points. You then substitute in one of the points and find c. Therefore

$$\begin{aligned} y &= \tfrac{11}{3}x + c \\ 7 &= \tfrac{11}{3} \times 6 + c \\ -15 &= c. \end{aligned}$$

Therefore the line is $y = \frac{11}{3}x - 15$. This can (more elegantly) be multiplied by 3 and rearranged to yield $11x - 3y - 45 = 0$.

Perpendicular Lines

Two lines are perpendicular if their gradients (m_1 and m_2) satisfy $m_1 \times m_2 = -1$. Therefore if a line has gradient 10, then the perpendicular has gradient $-\frac{1}{10}$. In general to find the gradient of the perpendicular you "flip over and change the sign". Best seen by a few examples:

Gradient	Perpendicular Gradient
2	$-\frac{1}{2}$
-4	$\frac{1}{4}$
$\frac{2}{3}$	$-\frac{3}{2}$
$-\frac{6}{7}$	$\frac{7}{6}$